The Astrophysical Journal, 549:72–75, 2001 March 1

CAN HEAVY WIMPs BE CAPTURED BY THE EARTH?

Andrew Gould and S. M. Khairul Alam
Department of Astronomy, Ohio State University, Columbus, OH 43210;
gould@astronomy.ohio-state.edu, alam@astronomy.ohio-state.edu
Received 1999 November 15; accepted 2000 October 24

ABSTRACT

If weakly interacting massive particles (WIMPs) in bound solar orbits are systematically driven into the Sun by solar system resonances (as Farinella and coworkers have shown is the case for many Earth-crossing asteroids), then the capture of high-mass WIMPs by the Earth would be affected dramatically because high-mass WIMPs are captured primarily from bound orbits. WIMP capture would be eliminated for $M_x > 630$ GeV and would be highly suppressed for $M_x \gtrsim 150$ GeV. Annihilation of captured WIMPs and anti-WIMPs is expected to give rise to neutrinos coming from the Earth's center. The absence of such a neutrino signal has been used to place limits on WIMP parameters. At present, one does not know whether typical WIMP orbits are in fact affected by these resonances. Until this question is investigated and resolved, one must (conservatively) assume that they are. Hence, limits on high-mass WIMP parameters are significantly weaker than was previously believed.

Subject headings: minor planets, asteroids — dark matter — elementary particles — Galaxy: halo

1. INTRODUCTION

If weakly interacting massive particles (WIMPs) comprise the dark matter, they would have a local density $\rho_x \sim 0.3$ GeV cm^{-3}. They would be potentially detectable in several ways: directly in underground detectors, from WIMP–anti-WIMP annihilations in the Galactic halo, and indirectly from neutrinos produced by annihilations of WIMPs captured by the Earth and the Sun (e.g., Jungman, Kamionkowski, & Griest 1996).

WIMP capture by the Sun was first discussed in the context of a proposed WIMP solution to the solar neutrino problem (Press & Spergel 1985; Faulkner & Gilliland 1985). Subsequently, several workers realized that if both WIMPs and anti-WIMPs were captured by the Sun (Silk, Olive, & Srednicki 1985; Gaisser, Steigman, & Tilav 1986; Srednicki, Olive, & Silk 1987; Griest & Seckel 1987) or Earth (Freese 1986; Krauss, Srednicki, & Wilczek 1986; Gaisser et al. 1986), they might annihilate into neutrinos, which would then pass directly through these bodies and into neutrino detectors near the surface of the Earth. The failure to detect neutrinos with relevant energies ($E_\nu \sim M_x c^2/2$) coming from the centers of the Sun and Earth could then place constraints on the WIMP mass M_x and cross section σ_x (e.g., Bottino et al. 1999).

Because the Sun's gravitational potential is much deeper than the Earth's, it initially appeared as though the Sun would be so much more effective at capture that the annihilation signal from the Sun would always be much larger than from the Earth. However, Gould (1987) showed that there were "resonant enhancements" in capture by the Earth whenever the WIMP mass M_x was close to the mass $M_{\rm A}$ of an element A that is common in the Earth. Specifically, for hard-sphere cross sections $\sigma_{x{\rm A}}$, the capture rate is given by (see also Gould 1992)

$$C = \frac{N_{\rm A}\sigma_{x{\rm A}}}{\beta_-}\int_0^{v_{\rm cut}} d^3u(v_{\rm cut}^2 - u^2)\,\frac{f(\boldsymbol{u})}{u}\,, \qquad (1)$$

where

$$v_{\rm cut}^2 \equiv \beta_- v_{\rm esc}^2\,, \quad \beta_\pm \equiv \frac{4M_x M_{\rm A}}{(M_x \pm M_{\rm A})^2}\,, \qquad (2)$$

$f(\boldsymbol{u})$ is the distribution of velocities $\boldsymbol{u}$ of WIMPs relative to the Earth in the terrestrial neighborhood but away from the Earth's potential, $N_{\rm A}$ is the total number of atoms A in the Earth, and $v_{\rm esc} \sim 13$ km s^{-1} is the escape velocity at the position of the atoms. In principle, one should allow for a range of escape velocities of the atoms, but for the case of iron in the Earth (of interest here), Gould (1987) showed that the error induced by simply using the average escape velocity is small.

In the present context, it is very important to note that $v_{\rm cut}$ is the maximum velocity for which an ambient WIMP can be captured by the Earth. If $M_{\rm A} \sim M_x$, then $\beta_- \gg 1$ and so $v_{\rm cut} \gg v_{\rm esc}$. In this case, capture is dominated by WIMPs that have velocities typical of the halo $v_h \sim 300$ km s^{-1}. This is the source of the "resonant enhancements" mentioned above: over the whole mass range 12 GeV $\lesssim M_x \lesssim$ 65 GeV, WIMP capture is dominated by resonances with elements common in the Earth, oxygen, magnesium, silicon, and iron. See Figure 2 of Gould (1987). For masses $M_x \gtrsim$ 65 GeV, WIMP capture is overwhelmingly due to the "tail" of the iron resonance. If the velocity distribution at low velocities could be approximated as a constant $f(\boldsymbol{u}) \simeq f(0)$, then the capture rate would take the form

$$C_0 = \frac{\pi N_{\rm A}\sigma_{x{\rm A}}\, f(0)v_{\rm cut}^4}{\beta_-} \to 4\pi N_{\rm A}\sigma_{x{\rm A}}\, f(0)v_{\rm esc}^4\,\frac{M_{\rm A}}{M_x} \qquad (3)$$

[assuming $f(\boldsymbol{u}) = f(0)$ for $u < v_{\rm cut}$]. Thus in the high-mass limit (evaluated after the arrow), the capture rate would fall as $C_0 \propto M_x^{-2}$: one power of M_x appears explicitly and the other is implicit in $f(\boldsymbol{u})$ for fixed ρ_x.

Gould (1988) showed that, unfortunately, the high-mass capture rate might not be so simple. If one restricts attention to WIMPs that are *not* bound to the Sun, then

$$f_{\rm unbound}(\boldsymbol{u}) = 0 \text{ for } |\boldsymbol{u} + \boldsymbol{v}_\oplus| \le 2^{1/2}v_\oplus\,, \qquad (4)$$

where $\boldsymbol{v}_\oplus$ is the velocity of the Earth. To evaluate capture in the high-mass regime properly, it is therefore necessary to evaluate the velocity distribution of bound as well as unbound WIMPs. In particular, if there were no bound WIMPs, then there would be no WIMP capture at all for WIMPs with $v_{\rm cut} \le (2^{1/2} - 1)v_\oplus$, and WIMP capture would be highly suppressed for WIMPs with $v_{\rm cut} \lesssim v_\oplus$.

These thresholds correspond to $M_x \sim 320$ GeV and $M_x \sim 120$ GeV, respectively.

Gould (1991) argued from detailed balance that regardless of the initial WIMP velocity distribution at the time of the formation of the solar system, $f_{\rm bound}(\boldsymbol{u})$ would be driven toward $f_\odot(0)$, the low-velocity limit of the velocity distribution in the solar neighborhood but away from the solar potential,

$$f_{\rm bound}(\boldsymbol{u}) \to f_\odot(0) \tag{5}$$

(Gould 1991). He found that the characteristic time for the evolution of the velocity distribution is less than the age of the solar system for $u \lesssim v_\oplus$ (Fig. 3 of Gould 1991). While these arguments still left indeterminate the bound WIMP distribution for $u \gtrsim v_\oplus$, in practice the capture rate would not be seriously affected for any mass even if all these orbits were empty. Hence Gould's (1991) arguments appeared to resolve the problem of WIMP capture for any M_x, $\sigma_{x\rm A}$, and Galactic WIMP distribution.

From the viewpoint of WIMP kinematics (the basic viewpoint adopted in this paper), the WIMP mass M_x is simply a parameter and can assume any value. However, the relevance of our results depends critically on the viability of WIMP models with masses $M_x \gtrsim 150$ GeV and especially $M_x > 630$ GeV, where the effects we describe become significant. When WIMPs were first hypothesized, considerations of known particle physics focused attention on relatively low masses O(10 GeV). As severe experimental limits were placed on these, a broader range of particle physics models were considered, the most popular being the mimimal supersymmetric standard model (MSSM). As summarized by Ellis (2000), there are strong but not absolutely secure limits on light WIMPs, $M_x \lesssim 50$ GeV. An argument due to Olive & Srednicki (1989, 1991) seemed to indicate that WIMPs could not be viable dark matter candidates unless they were $M_x \lesssim 300$ GeV, but a loophole found by Ellis, Falk, & Olive (1998) raises this to $M_x \lesssim 600$ GeV. Thus the entire range of masses explored in this paper is considered relevant even within the MSSM. Of course, it is also possible that WIMPs are not MSSM particles, in which case the masses could be even higher. Ultimately, unitarity considerations place model-independent limits on the WIMP mass, $M_x \lesssim 3$ TeV (Griest, Kamionkowski, & Turner 1990; Griest & Kamionkowski 1990; Jungman et al. 1996).

2. NEW DEVELOPMENTS

There have been two new developments since 1991 that challenge the seemingly closed case described in the previous section. First, Farinella et al. (1994) have shown by direct numerical integration that a large fraction of Earth-crossing asteroids are systematically driven into the Sun by various solar system resonances. Gladman et al. (1997) and Migliorini et al. (1998) have further studied this problem and generally confirmed the initial results. These solar collisions occur on Myr timescales, several orders of magnitude faster than the characteristic diffusion times evaluated by Gould (1991). If WIMPs were, like asteroids, also driven into the Sun, they would be captured by the Sun and hence would be unavailable for capture by the Earth.

Second, Damour & Krauss (1998, 1999) have shown that WIMPs captured by the outer layers of the Sun into highly eccentric orbits could evolve into non–Sun-crossing orbits before they again collided with solar nuclei. If so, WIMPs on these eccentric orbits could substantially increase the number of low-velocity WIMPs in the solar neighborhood and so could dramatically increase the capture rate in the mass range 60 GeV $\lesssim M_x \lesssim$ 130 GeV (Bergström et al. 1999). Below 60 GeV, WIMP capture is dominated by the iron resonance while above 130 GeV, $v_{\rm cut} > v_\oplus$, the minimum speed relative to the Earth for WIMPs on highly eccentric orbits.

Here we calculate the suppression factor for WIMP capture assuming that WIMPs are systematically driven into the Sun by resonances, relative to the rate produced by a uniform phase-space density as advocated by Gould (1991).

3. BASIC APPROACH

The central experimental fact that must guide our analysis is that to date no relevant neutrinos have been detected from the center of either the Earth or the Sun. The experiments therefore place upper limits on WIMPs. Thus, a conservative interpretation of these experiments requires that one assume that only those parts of velocity space are populated as can be justified based on very secure theoretical arguments. This perspective implies that we must assume that *all* bound WIMPs with Earth-crossing orbits are driven into the Sun within a few Myr. This includes primordial WIMPs, the gravitationally diffused WIMPs of Gould (1991), and the solar-collision WIMPs of Damour & Krauss (1998, 1999).

Before continuing, we wish to emphasize that it is by no means proved that all bound WIMPs are in fact driven into the Sun. The numerical integrations carried out to date (Farinella et al. 1994; Gladman et al. 1997; Migliorini et al. 1998) apply to a very special subclass of Earth-crossing orbits, namely, those of existing minor bodies. Near-Earth asteroids are believed to be transported from their reservoir in the asteroid belt by means of resonances. It therefore may not be surprising that their continued orbital evolution is dominated by resonances. The main population of Earth-crossing WIMPs, which acquire their orbits by quite different, nonresonant paths (Gould 1991; Damour & Krauss 1998, 1999), could be virtually unaffected by the resonances that drive asteroids into the Sun. Our viewpoint is simply that in the absence of proof that Earth-crossing WIMPs are not depopulated, one cannot place reliable upper limits on WIMPs from the failure to detect the annihilation signal that would be triggered by the capture of these Earth-crossing WIMPs.

4. ASSURED WIMP CAPTURE

We begin by adopting an ultraconservative view and assume that all bound WIMPs acquired by the solar system are immediately driven into the Sun. Then, for $v_{\rm cut} < (2^{1/2}+1)v_\oplus$, it is straightforward to show from equation (1) that

$$C_{\rm ultra} = \frac{2\pi N_{\rm A}\,\sigma_{x\rm A}\, f_\oplus(0)}{\beta_-} \times \int_{u=v_{\rm hole}-v_\oplus}^{v\rm cut} du^2 (v_{\rm cut}^2 - u^2) \times \left(1 - \frac{v_{\rm hole}^2 - v_\oplus^2 - u^2}{2uv_\oplus}\right), \tag{6}$$

where

$$v_{\rm hole} = 2^{1/2} v_\oplus \; . \qquad (7)$$

Hence, the ratio of the number of WIMPs captured under ultraconservative assumptions to the naive formula (3) is

$$\frac{C_{\rm ultra}}{C_0} = \left\{ \frac{1}{2}(1 - s^2) + \frac{v_{\rm cut}}{v_\oplus}\left[-t^2(1-s) + \frac{(1+t^2)(1-s^3)}{3} - \frac{1-s^5}{5} \right] \right\} \Theta(1-s) \; , \qquad (8)$$

where

$$s = \frac{v_{\rm hole} - v_\oplus}{v_{\rm cut}} = 0.41\,\frac{v_\oplus}{v_{\rm cut}} \; , \quad t = \frac{\sqrt{v_{\rm hole}^2 - v_\oplus^2}}{v_{\rm cut}} = \frac{v_\oplus}{v_{\rm cut}} \; , \qquad (9)$$

and Θ is a step function. The ratio $C_{\rm ultra}/C_0$ as a function of WIMP mass M_x is shown as a thin solid line in Figure 1.

Equation (6) is in fact too conservative. Gould (1991) showed that Jupiter-crossing orbits (including those that also cross the Earth's orbit) are populated from the reservoir of Galactic WIMPs on very short timescales. To an adequate approximation, the Jupiter-crossing orbits can be described as those with Earth-crossing velocities $\boldsymbol{u}$ constrained by $|\boldsymbol{u} + \boldsymbol{v}_\oplus| > [2(1 - a_\oplus/a_{\rm J})]^{1/2} v_\oplus$, where $a_{\rm J}/a_\oplus \simeq 5.2$ is the ratio of the orbital radii of Jupiter and the Earth. Hence the ratio of capture rates in this conservative (but not ultraconservative) framework is still given by equation (8) but with $v_{\rm hole}^2 \to 2(1 - a_\oplus/a_{\rm J}) v_\oplus^2$, and consequently

$$s = 0.27\,\frac{v_\oplus}{v_{\rm cut}} \; , \quad t = 0.78\,\frac{v_\oplus}{v_{\rm cut}} \; . \qquad (10)$$

This result is shown as a bold curve in Figure 1.

In principle one should take into account the loss of coherence in WIMP-nucleon interactions that occurs when the momentum transfer, q, becomes comparable to (or larger than) $\hbar$ divided by the radius of the nucleus, $R \sim 3.7$ fm for iron. The suppression due to loss of coherence is $\exp(-q^2 R^2/3\hbar^2)$ (Gould 1987). However, for WIMPs of mass M_x, the maximum momentum transfer that leads to capture is $q_{\rm max} = (M_{\rm Fe} M_x)^{1/2} v_{\rm cut}$. In the high-mass limit, $q_{\rm max} \to 2 M_{\rm Fe} v_{\rm esc}$. Hence, the most extreme suppression factor is $\exp[-(2 M_{\rm Fe} v_{\rm esc} R_{\rm Fe}/\hbar)^2/3] \sim 0.997$. We conclude that loss of coherence can be ignored. See also Gould (1987).

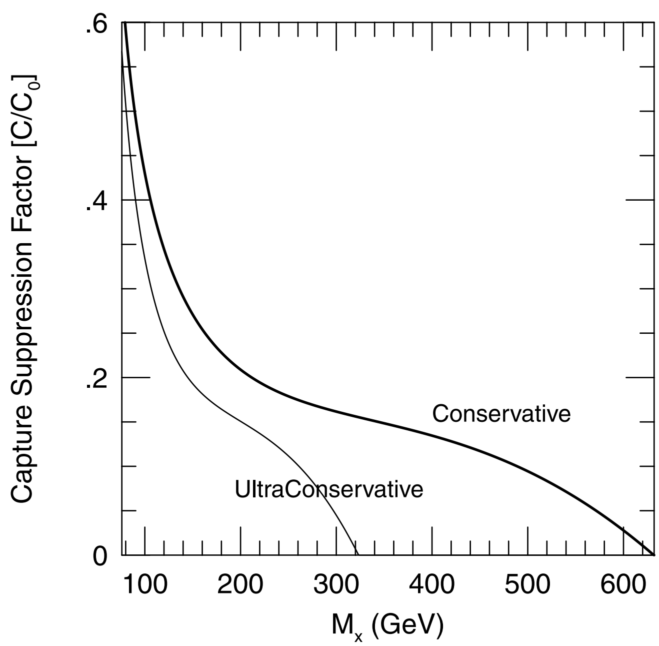

FIG. 1.—Conservative capture rates for WIMPs relative to the rate based on the naive assumption (Gould 1991) that the phase-space density of WIMPs bound to the Sun is similar to that of low-velocity unbound WIMPs. The thin solid curve shows the suppression factor under the ultraconservative assumption that all bound WIMPs are driven into the Sun on short timescales (as is true of many Earth-crossing asteroids). The bold curve results from the more realistic assumption that WIMPs on Jupiter-crossing (and Earth-crossing) orbits are repopulated faster than they can be driven into the Sun.

5. DISCUSSION

From Figure 1, we see that under the conservative assumption that WIMPs do not populate bound orbits (unless they are Jupiter-crossing), WIMP capture is highly suppressed for WIMP mass $M_x \gtrsim 150$ GeV and completely eliminated for $M_x > 630$ GeV. These results imply that, at present, the nondetection of neutrinos coming from the Earth's center cannot be used to place limits on WIMPs of mass $M_x > 630$ GeV. Moreover, for masses 75 GeV $\lesssim M_x \lesssim 630$ GeV, the limits must be softened relative to what would be obtained from equation (3).

It is quite possible that WIMPs are not generically driven into the Sun. The evidence that they are comes from numerical integration of asteroid orbits, and the latter could occupy a very special locus in parameter space. It will be necessary to integrate typical WIMP orbits to find out whether WIMPs survive longer than asteroids. In particular, these integrations should focus on the highly eccentric orbits for which Damour & Krauss (1998, 1999) predict a huge enhancement for $M_x \lesssim 130$ GeV. If typical WIMP orbits are found to survive substantially longer than asteroid orbits, then the limits derived from the naive calculations of Gould (1987, 1991) would become valid. It is even possible that the much stronger limits derived by Bergström et al. (1999) would apply.

We thank A. Quillen for pointing out the importance of recent work on asteroid orbits. This work was supported in part by grant AST 97-27520 from the NSF.

REFERENCES

Bergström, L., Damour, T., Edsjo, J., Krauss, L. M., & Ullio, P. 1999, J. High Energy Phys. (online), 09, 999

Bottino, A., Donato, F., Fornengo, N., & Scopel, S. 1999, Astropart. Phys., 10, 203

Damour, T., & Krauss, L. M. 1998, Phys. Rev. Lett., 81, 5726

———. 1999, Phys. Rev. D, 59, 063509

Ellis, J. 2000, Proc. 26th Int. Cosmic-Ray Conf., submitted (astro-ph/9911440)

Ellis, J., Falk, T., & Olive, K. 1998, Phys. Lett. B, 444, 367

Farinella, P., Froeschlé, Ch., Froeschlé, Cl., Gonczi, R., Hahn, G., Morbidelli, A., & Valsecchi, G. B. 1994, Nature, 371, 314

Faulkner, J., & Gilliland, R. L. 1985, ApJ, 299, 994

Freese, K. 1986, Phys. Lett. B, 167, 295

Gaisser, T. K., Steigman, G., & Tilav, S. 1986, Phys. Rev. D, 34, 2206

Gladman, B., et al. 1997, Science, 277, 197

Gould, A. 1987, ApJ, 321, 571

———. 1988, ApJ, 328, 919

———. 1991, ApJ, 368, 610

Gould, A. 1992, ApJ, 388, 338
Griest, K., & Kamionkowski, M. 1990, Phys. Rev. Lett., 64, 615
Griest, K., Kamionkowski, M., & Turner, M. S. 1990, Phys. Rev. D, 41, 3565
Griest, K., & Seckel, D. 1987, Nucl. Phys. B, 283, 681
Jungman, G., Kamionkowski, M., & Griest, K. 1996, Phys. Rep., 267, 195
Krauss, L. M., Srednicki, M., & Wilczek, F. 1986, Phys. Rev. D, 33, 2079
Migliorini, F., Michel, P., Morbidelli, A., Nesvorný, D., & Zappalà, V. 1998, Science, 281, 2022
Olive, K. A., & Srednicki, M. 1989, Phys. Lett. B, 230, 78
———. 1991, Nucl. Phys. B, 355, 208
Press, W. H., & Spergel, D. N. 1985, ApJ, 296, 679
Silk, J., Olive, K., & Srednicki, M. 1985, Phys. Rev. Lett., 55, 257
Srednicki, M., Olive, K A., & Silk, J. 1987, Nucl. Phys. B, 279, 804